Part 4

The project argues that “the Manhattan distances are consistent in gridworlds in which the agent can move only in the four main compass directions.” Prove that this is indeed the case.

Furthermore, it is argued that “The h-values hnew(s) ... are not only admissible but also consistent.” Prove that Adaptive A\* leaves initially consistent h-values consistent even if action costs can increase.

A heuristic h(n) is consistent if, for every node n and every successor n’ of n

generated by any action a, the estimated cost of reaching the goal from n is no

greater than the step cost of getting to n’ plus the estimated cost of reaching the

goal from n’. In other words, the heuristic function must underestimate the actual

path cost and the following triangular inequality must hold.

h(n) ≤ step cost (to neighbor n’) + h(n’)

Manhattan distance is calculated by adding the horizontal and the vertical path.

In the gridworld, Manhattan distance is always the same and it is the fastest path

possible to reach the goal as the agent can move only up, down, left, or right.

Here, the agent cannot move diagonally and so the agent will always find the

shortest path. So, Manhattan distance is consistent. If the agent can go diagonally,

the heuristic can overestimate the distance to reach the target.

To Prove that Adaptive A\* is consistent:

Since, h-values are consistent, and h(s) follows Manhattan Heuristics.

hnew (s) = g(sgoal) – g(s). 🡪 1

c(s,a,s’) – Step cost of going from s to s’ using action a is ONE.

The triangular inequality says

h(s) ≤ h(s’) + c(s,a,s’)

We have to prove hnew (s) ≤ hnew (s’) + c(s,a,s’)

Substituting (from 1) hnew (s) = g(sgoal) – g(s) and hnew (s’) = g(sgoal) – g(s’) above ,

We get g(sgoal) – g(s) ≤ g(sgoal) – g(s’) + c(s,a,s’)

Simplifying we get

=> g(s) ≥ g(s’) – c(s,a,s’)

This is TRUE because:

* In Manhattan Heuristics c(s,a,s’) is always ONE
* If g(s’) is smaller than g(s), subtracting ONE from it will make it still smaller by ONE
* If g(s’) is greater than g(s), subtracting ONE from it will make them equal

So, h-values hnew (s) are consistent.

Now, to prove that Adaptive A\*leaves initially consistent h-values consistent even if action costs can increase:

Let us again consider the triangular inequality

hnew(s) ≤ hnew(s’) + c(s,a,s’)

Let there be a cost increase and c(s,a,s’) be the cost before and c’(s,a,s’) be the cost

after the increase.

Now hnew(s) ≤ hnew(s’) + c(s,a,s’) ≤ hnew(s’) + c’(s,a,s’)

(since c’(s,a,s’) is more than c(s,a,s’) )

So, we can see that the heuristic is consistent with action cost increase.